THE UNREASONABLE EFFECTIVENESS OF SCIENTIFIC METHOD

HOW ARE WE TO EXPLAIN WIGNER’S EXAMPLES?

WIGNER’S “MIRACLE”

- In 1960, Eugene Wigner published a paper called “The Unreasonable Effectiveness of Mathematics in the Natural Sciences”.
- He concluded his paper with the words”
- “[the] miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we can neither understand nor deserve.”
- What was he getting at?

WIGNER'S PROBLEM

- Wigner believed that, in many cases, mathematics, or the language of mathematics, had been "miraculously" appropriate or useful or effective in physics.
- He claimed that, if we look at the history of physics we find many cases in which the theories advanced by scientists had given out more knowledge than we, or observations, had initially put in to them.
- Epistemologically, it was as if we had poured one litre of water in and miraculously got two litres out.
- How had this happened?
- Wigner's thesis was that there was something about the language of mathematics that was responsible for this "miracle".

WIGNER'S EXAMPLES

- One example that Wigner mentions is Newton's inverse square law of gravitation.
- Wigner says Newton first put forward this law on the basis of "very scanty" observations, and his empirical data were not so accurate as to irresistibly suggest an inverse square law to Newton. (They were only accurate to about 4%).
- But subsequent observations confirmed the accuracy of Newton's inverse square law to (Wigner reports) an accuracy of one ten-thousandth of a percent.
- (A similar point might be made about Coulomb's law – initially based on observations accurate to about 4%, but now confirmed to have an accuracy of one part in at least a billion.)

WIGNER’S EXAMPLES

- Wigner’s second and third examples comes from quantum mechanics.
- However, we will not concern ourselves with them here.

STEINER AND MAXWELL’S EQUATIONS

- Mark Steiner in *The Applicability of Mathematics as a Philosophical Problem* gives the example of Maxwell’s theory of electromagnetism.
- Maxwell noted that Gauss’ laws for electricity and for magnetism, Faraday’s law and Ampere’s law jointly violated the principle of the conservation of electric charge. To deal with the difficulty, Maxwell carried out a modification to Ampere’s law. The modification was not made on the basis of observational evidence, but was rather made on the basis of what has been described as “mathematical analogy”, “aesthetic considerations”, or even “poetic justice”. This modified version of Ampere’s law, together with the other above-mentioned laws, resulted in what we know now as “Maxwell’s Equations”.

MAXWELL'S EQUATIONS

- These equations led to the prediction of electromagnetic radiation, to the hypothesis that such radiation would travel at the speed of light, that light was itself a form of electromagnetic radiation, and to the prediction of the existence of radio waves.
- These predictions all turned out to be correct. The puzzling or "miraculous" thing is that these correct predictions followed from equations initially formulated, at least in part, on non-empirical grounds of "mathematical analogy", aesthetics etc.

WHAT THE EXAMPLES (SEEM TO) SHOW

- In all the cases, the amount of knowledge that followed from the equations was surprisingly (even “miraculously”) greater than the empirical input on which they were based.
- Why?
- According to Wigner (and Steiner, Colyvan, Arianrhod and others) there is something about the **language of mathematics** that seems to be responsible for this surprising increase.

IS THERE A GENUINE PROBLEM HERE?

- Not all authors have accepted that there is a genuine problem here.
- Although I’ll not go in to in the matter here, it seems to me that there is a real puzzle here.

A VERY SIMPLE EXAMPLE

- I wish to approach the problem by considering a very simple example: rather simpler than any of the cases discussed by Wigner or others.
- The case I wish to discuss is Newton’s first law motion.
- It will be argued that this case possesses the features that puzzle Wigner, although perhaps in a less spectacular form.
- An examination of this simple case will suggest a strategy for dealing with the more complex cases.

NEWTON’S FIRST LAW

- Let us begin by considering a statement of Newton’s first law:

- A body at rest remains at rest or moving at uniform velocity in a straight line unless acted upon by some (external, unbalanced) force.

- There are two features of this law that are worth immediately noting.
- First: it is about bodies not acted upon by any force. But no one has ever (directly) observed such a body. Certainly, Newton had never observed such a body.

NEWTON'S FIRST LAW

- Secondly: it asserts that these bodies remain at rest or moving in a straight line.
- But: no one has ever observed a perfectly straight line.
- A "straight line" is plausibly a mathematical/geometrical concept.
- So, there is a sense in which the law would not seem to be at least directly based on observation, and it is expressed in mathematical or geometrical terms.
- And yet, it has been very highly empirically successful.

NEWTON'S FIRST LAW

- Observations made after Newton confirmed that, at least to a very high degree of accuracy, objects not acted on by any force do move in straight lines.
- As we developed more accurate techniques of measurement, our obtained results (at least for a while) came closer and closer to saying objects moved in straight lines.
- In this respect, Newton's first law had success similar to one of Wigner's examples: Newton's law of gravitation. According to Wigner, this was initially formulated on the basis of scanty observations, but continued to seem to get things right even as our observations became more and more accurate.
- Newton's law asks us to suppose something in nature has the properties of a mathematical entity, and despite the scantiness of the evidence for this, it leads to impressive subsequent success.
- So, this does seem to be a case of the same sort as that which puzzles Wigner.

IS THERE SOMETHING PUZZLING HERE?

- It seems there is something here that requires explanation.
- The observations available to Newton were *compatible* with saying the objects moved in a perfectly straight line.
- But they were compatible with many other hypotheses as well, (Various curves that were, at the time, observationally indistinguishable from a straight line.)
- But Newton chose a straight line. And subsequent observations confirmed the correctness of that hypothesis.
- How are we to explain this?

SOME POSSIBLE EXPLANATIONS

- Is the subsequent success of Newton's hypothesis to be explained by pointing out the **mathematical**/**geometrical** nature of a concept that appears centrally in the statement of the law (a straight line?)
- That clearly won't do.
- There are very many other curves that Newton could have said the objects followed which would have resulted in laws that did not receive subsequent confirmation.
- But these other curves are every bit as mathematical/geometrical as a straight line.
- Mathematicalness, by itself is clearly not sufficient for the general type of effectiveness Wigner has in mind.

IS MATHEMATICAL LANGUAGE NECESSARY?

- But is mathematical language perhaps **necessary** for the type of success with which Wigner is concerned?

IS MATHEMATICAL LANGUAGE NECESSARY? THE EXAMPLE OF EVOLUTION.

- It will be argued that mathematical language is not necessarily a feature of all theories that are capable of giving us in this sense an increase in our knowledge.
- One example is the theory of evolution.
- The theory of evolution (without genetics) can be derived by a plausible argument assuming only population growth, chance variation, heritability and limited resources. (As far as I am aware there is not a single mathematical equation in *The Origin of Species*.)
- But it leads to the prediction that evolution occurs by means of steps small enough to have occurred by chance, and each of which increases fitness.
- And this can, perhaps with other assumptions, lead to numerous novel predictions, e.g about transitional forms, the fossil record, and the non-existence of irreducible complexity.

IS MATHEMATICAL LANGUAGE NECESSARY

- Another example comes from history.
- R.G. Collingwood.
- Collingwood saw at least a part of history as imaginatively reconstructing the thoughts of people who lived a long time ago.
- He illustrates this with an example concerning Hadrian’s Wall.
- Rooms had been found at various locations on the wall.
- Before Collingwood, historians believed they might be resting rooms for guards.

COLLINGWOOD AND HADRIAN’S WALL

- Collingwood reports that “imaginatively reconstructing” the thoughts of those who had constructed the wall led him to the hypothesis that the purpose of the rooms was actually to observe the land beyond the wall.
- And this led him to a prediction: that more of these rooms ought to be discovered at particularly high or prominent points in the wall, or points commanding a good view of the land beyond.
- He reports that this prediction was in fact confirmed.
- This would seem to be a case of a hypothesis yielding more knowledge than was put in to it.
- But there would not seem to be anything that it would be remotely plausible to call “mathematical language” playing any role here.

CONCLUSION

- There are some circumstances in which it would surely have to be the case that a surprisingly successful theory was expressed in mathematical language. This would, plausibly, have to be the case if our actually obtained observations became, over time, **increasingly close** to those predicted by the theory. (The very concept of **closeness** seems to require some sort of “mathematicalness”)
- But, mathematical language seems to be neither sufficient, nor necessary, for the type of success considered more generally with which Wigner is concerned.
- (Darwin, Collingwood)

SIMPLICITY

- If success is not due to “mathematicalness”, what might be responsible?
- Simplicity?
- Wigner does not explicitly reject the idea that success is due to simplicity, but does imply simplicity is not the right concept to which to appeal.
- Wigner’s arguments against simplicity are I think unconvincing. (He claims our theories are not simple)
- Here I will not discuss Wigner’s arguments in full detail, but will simply note what I take to be an obvious point – the mathematical ideas that have been of interest to us are surely far simpler than most possible mathematical ideas.

A MORE DEFINITE SUGGESTION

- We noted an obvious feature of Newton's first law:
- It seems to be a lot simpler than the alternatives.
- But we seem to be able to be a bit more specific than that.
- We can say the following things about Newton's first law:
- It has kinetic energy and momentum being **conserved**.
- It postulates a **symmetry**. Or: it says certain properties are **invariant** across time.
- Might these cluster of features of theories – **conservation**, **symmetry**, **invariance** – be the ones that are responsible for the type of success with which Wigner is concerned?

CONSERVATION LAWS

- At least two of the other examples of surprising success seem to follow from the hypothesis that certain quantities are **conserved**.

- Inverse square laws of force (gravity, electrical force)

- Maxwell’s equations for electromagnetism.

INVERSE SQUARE LAWS AND CONSERVATION

- Inverse square laws can be derived from assumptions about conservation.
- Suppose, for example, gravitational or electrical force is thought of as being due to something like a gas or fluid emanating the source of the force. Then, if the total quantity of the gas is conserved as it moves away from the mass, the density of the gas will obey an inverse square law.
- (If force is proportional to density, so will the force obey an inverse square law.)

MAXWELL'S EQUATIONS AND CONSERVATION

- Maxwell found that Gauss' laws for electricity and magnetism, Ampere's law and Faraday's law together implied that electric charge was **not** conserved.
- Maxwell sought to modify the existing theories of electricity and magnetism so that electrical charge was conserved.
- This led to the formulation of what we now call Maxwell's equations for electromagnetism.
- Maxwell's equations, as we noted, led to numerous surprising successes.

CONSERVATION, SYMMETRY, INVARIANCE

- In these examples (at least as we have presented them) it is the maintenance of conservation laws that have played the crucial role.
- But there are other ways of looking at the examples.
- To say a quantity is conserved is to say it is invariant over time, or that its magnitude is symmetrical with respect to position in time.
- So, our discussion so far suggests it could be conservation, symmetry or invariance that is responsible for the surprising success.

THE NON-MATHEMATICAL EXAMPLES

- Let us now consider the non-mathematical examples.
- Can these examples of surprising success be given an explanation similar to the explanation given for the mathematical cases?
- It will be argued that they can be.
- More specifically, it will be argued that it is something like invariance or symmetry that seems to play a crucial role in these examples of success.

EVOLUTION

- We can see the theory of evolution as asserting that slow, gradual changes in species occur. When these small changes occur over very long periods of time, the eventual result is the appearance of a new species.
- The fact that this slow process of change occurs is a result of the following factors remaining invariant over time.
- (1) Limited resources.
- (2) More offspring than parents – so that if unchecked population would increase exponentially.
- (3) Slight variation amongst the members of any generation.
- (4) Variable ability to survive and (therefore) natural selection.

EVOLUTION

- The important point is that (1) – (4) all remain true for any point in time (on Earth, over the last few billion years)
- They are forms of invariance, or symmetry.
- From (1) – (4) the hypothesis of gradual change follows – which also leads to a variety of novel or surprising predictions.
- So, the assumption that certain processes or states of affairs are **invariant**, leads to a range of surprising or risky predictions, or to new knowledge.
- (Uniformitarianism in geology?)

COLLINGWOOD’S OBSERVATION ROOMS

- Let us now consider the example of Collingwood’s guard-rooms on Hadrian’s wall.
- Collingwood said he was led to the hypothesis that the rooms were actually for observation by imaginatively putting himself in the same frame of mind as the Ancient Romans.
- (If I were in the same position as an Ancient Roman, why would I put rooms on the wall?)
- But what is Collingwood assuming here?

COLLINGWOOD'S ASSUMPTIONS

- It would seem that Collingwood – at least if he believes this method will lead to success – must be assuming that in broad terms an Ancient Roman is going to have the same concerns, interests, motives and general beliefs as Collingwood himself would have if he were placed in the same situation.
- So, Collingwood would seem to be making the more general assumption that, in some sense which it is difficult to make precise, he, the Ancient Romans, and presumably all other members of humanity are more or less the same "on the inside": we all have at least broadly similar concerns and psychological propensities. (Similar enough for this method to lead to true predictions)

COLLINGWOOD’S ASSUMPTION

- But if this is so then Collingwood is making something that can reasonably be called an invariance or symmetry assumption.
- He is assuming that in broad terms at least something like “psychological concerns or propensities” remain the same no matter what mind we are in.
- The psychological concerns or propensities remain **invariant** as we go from one mind or another
- (Definition of symmetry: A property P of a system S exhibits symmetry iff P remains unchanged when S is subject to some transformation)

SUMMARY

- Let us now summarise the results so far.
- We have looked at a range of cases of “more knowledge coming out than was initially put in”.
- Some are mathematical, others are not. “Mathematicalness” is not what they all have in common.
- However, all involve an assumption of invariance, or something like it.
- Hypothesis: it is not “mathematicalness” but the assumption of **invariance**, or something like it, that in essential to producing the type of success with which Wigner was concerned.